STUDIA UBB GEOGRAPHIA, LIX, 2, 20154, pp. 77-86
(RECOMMENDED CITATION)

IN SEARCH OF IDEAL FORM- RATIO OF TRIANGULAR CHANNEL

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ABSTRACT. – In Search of Ideal Form-Ratio of Triangular Channel. Cross-sectional form of a natural channel is a two dimensional variable which is thoroughly studied by scholars from different fields on natural sciences like hydrology, geology, geomorphology, etc. Average river channels tend to develop their channel-cross sectional form in a way to produce an approximate equilibrium between the channel and the water and sediment it transport. But how far it is deviated from the ideal cross-sectional form can only be determined by knowing the ideal form which was calculated by Hickin for *rectangular channel.* This ideal cross-sectional form of 'maximum efficiency' is virtually a theoretical one and attaining of which the river transports its water and load with least friction with its bed. '*Ideal form ratio'* provides numerical tools for *triangular channel* to determine the degree of deviation of a cross-sectional form from that of an ideal one.

Keywords: *channel asymmetry, cross-sectional area, hydraulic radius, ideal form ratio, wetted perimeter.*

1. INTRODUCTION

The cross-sectional form of natural stream channel is characteristically irregular in outline and locally variable (Knighton, 1998). Commonly two groups of aspects are considered to describe channel cross-sectional form – i) channel size and ii) channel shape. The first group includes width (w), mean depth (d), cross-sectional area (A), wetted perimeter (P), hydraulic radius (R), maximum depth (d_{max}), and bed width (w_b) while the second group of variables are width: depth ratio (w/d), maximum depth: mean depth ratio (d_{max}/d), channel asymmetry, bank slope (Ø) and bed topography. About 9/10 meandering channels cross-sections are asymmetric (Leopold and Wolman, 1960) and symmetry is assumed to be found at cross-overs (Knighton, 1981) although cross-sectional asymmetry is found in so-called straight channels (Einstine and Shen, 1964; Keller, 1972). For the study of channel's cross-sectional form, it is indispensable to know the ratio and Hickin (2004) calculated it for rectangular channel. The present paper is related to the cross-sectional shape of triangular channel and aims to formulate indices on channel form to determine the magnitude of deviation of a cross-sectional shape from the ideal one.

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2. EVALUATION OF EXISTING MEASURES

Asymmetry of natural features has been studied from the view point of processes of action (Kranck, 1972; Clifton, 1976) and shape (Sharp, 1963; Tanner, 1967; Kennedy, 1976). The cross-sectional form of a river is primarily adjusted by bank erosion and lateral channel migration (Simon and Castro, 2004) and represents dominant measures of channel response (Simon, 1992; Simon and Darby, 1997). Channel shape is rarely explained in a single measure like width: depth ratio (w/d), maximum depth: mean depth ratio (d_{max}/d), channel asymmetry, bank slope (Ø), etc. **Width: mean-depth ratio** (w/d) does not define cross-sectional shape (Hey, 1978) yet it is a widely used index. **Maximum depth: mean depth ratio** (d_{max}/d) is another measure which gives no better expression than that of Width: mean-depth ratio (w/d).

Form ratio w/d or w/d_{max} (Schumm, 1960) and section asymmetry a_l /a_r (Milne, 1979) regarding cross-sectional form give an absolute measure. These measures do not tell about the quality of the ratio i.e. how far the value is deviated from ideal one. That is why ideal value of form ratio of triangular channels is to be known.

2.1. Channel asymmetry: Channel asymmetry is a measure of the degree of asymmetry of the two halves of the cross-sectional area demarcated by the central line drawn vertically from the mid-width point of the channel. Three channel asymmetry indices were defined (Fig. 1) by Knighton (1981).

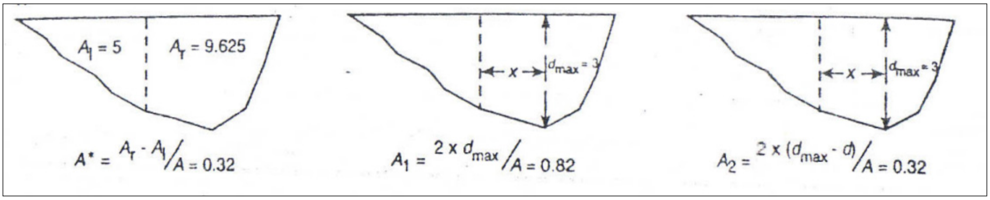


Fig. 1. Definitions of Asymmetry Indices
Source: Knighton (1998)

1. $A^* = (A_r - A_l) / A$

Where A_r and A_l are the cross-sectional areas, respectively, to the right and left of the channel centerline, A (= A_r + A_l) is the total area. Zero (0) value of the index indicate perfect symmetrical channel cross-sectional form while +1 and -1 indicates the maximumlimits of asymmetry to the left and right respectively.

2. $A_1 = (2x\, d_{max}) / A$

The equation is derived as (2x/w) (d_{max}/d). Now, w × d is replaced by A. x is the distance from the centerline to the centroid of maximum depth, $d_{max}$is maximum depth, and d is mean depth.

3. $A_2 = 2x(d_{max} - d) / A$

The equation is simplified form of $(2x/w)\ (d_{max} - d/d)$. Here depth of the channel is given appropriate weightage. To show the comparison amongst asymmetry in terms of width and depth of channels of equal area, Knighton (1981) used the term 'vertical asymmetry'. For knowing degree of vertical asymmetry, calculation of ideal depth of a channel of given cross-sectional area is precondition. Ideal form ratio provides that tool.

2.2. Bank slopes: Bank slopes measured in degrees are essentially concave in nature with rare variations like rectilinear or convex. Banks of Type- C and Type- B (Fig. 2) channels (Knighton, 1998) are rectilinear (Knighton 1998) while Type- A is more concave in nature. Degree of curvature (concavity or convexity) of a bank slope may be measured by the ratio of arc length to the angle in between lines of radii joining two ends of that arc. There is a opposite relation between the ratio and the degree of curvature of slope.

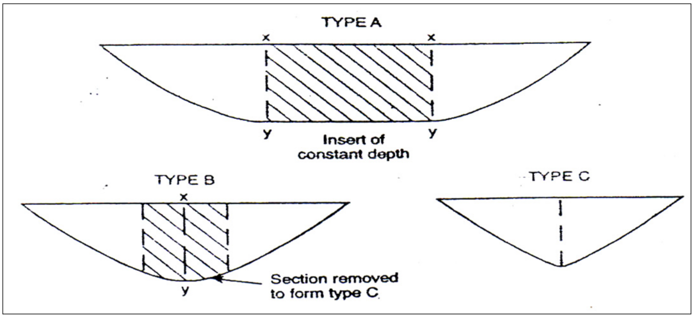


Fig. 2. Different Type of Cross-sectional form
Source: Knighton (1998)

2.3. Bed Topography: The last but not the least consideration of the channel cross-sectional form is its **bed topography**. Riffles-pool, step-pool, ripple, bar, shoal etc gives every channel a unique cross-sectional form which is beyond the capacity of a uniform measure to explain the shape as it is.

3. IDEAL CROSS-SECTIONAL FORM

The conventional belief of the v-shaped cross-sectional form of the rivers is far from the reality (Sen, 1993). Circular (Leopold et al, 1969) and parabolic (Lane, 1955) forms are also theoretical. Rather trapezoidal form represents the reality (Sen, 1993). But all these forms, whether theoretical or practical, are not obvious for all the channels or the entire reach of the same channel. The straight course of a river is impossible (Leopold and Langbein, 1966) which makes another impossibility of uniformity of cross-sectional form of the channel. Width increases faster than depth in downstream and cross-sectional form becomes increasingly rectangular (Sen, 1993). But sometimes the opposite is also the reality (Knighton, 1998; Das, 2013).

The conditions of efficiency of the cross-sectional characteristics of the channels are closely related to their capacity for allowance of maxium flow. Maximum flow (water + sediment load) is only possible when the cross-sectional form attains the semi-circular or parabolic shape (Knighton, 1998) or equilateral-triangular. These shapes generate the minimum turbulence and shear stress hence the 'most efficient' channels. Thus this ideal condition of channel form is considered as the 'best convayence characteristics' (Crickmay, 1974). Arcs of circles and parabolic curves may be simple devices for the explanation of the channel cross-sectional form. The relationship between channel form and processes operating in the channels has been studied as hydraulic geometry of the stream channels by Leopold and Maddock (1953), Wolman (1955), Leopold and Miller (1956) and others. They computed cross-sectional forms in terms of mean-depth (d) and width (w) in terms of hydraulic parameter discharge (Q).

$$w = {}_{a}Q^{b}$$
$$d = {}_{c}Q^{f}$$

Different average exponent values for ^{b} and ^{f} of different rivers have been calculated by Leopold and Maddock (1953), Wolman (1955), Leopold and Miller, (1956), Lewis (1969), Wilcock (1971) and Harvey (1975).

Manning (1891) flow resistance equation $v= k\,(R^{2/3}S^{1/2})/n$ (Knighton, 1998) or $v= (1/n)\ R_h^{2/3}S_b^{1/2}$(Simon and Castro, 2003) suggests that channel form and flow resistance determine the velocity of a river. To built up his equation, Manning utilised seven different open channel flow equations and the findings were field tested (Chaw, 1959; Fischenich, 2000). In this equation, v = velocity of flow, R or R_h = hydraulic radius of the channel, S or S_b = slope, and n = manning resistance co-efficient. With given volume, velocity is proportional to hydraulic radius ($R_h^{2/3}$) and slope ($S^{1/2}$) but inversly proportional to shear resistance. Valley slope and floodplain slope is generally greater than channel slope (Simon and Castro, 2003). If all other variables of the equation are constant, the equation implies that the velocity over flood plain is greater than in the channel. But reality is opposite because of- i) Roughness, hence shear strength over flood plain is much greater than in the channel; ii) Hydraulic radius is lesser over floodplain.

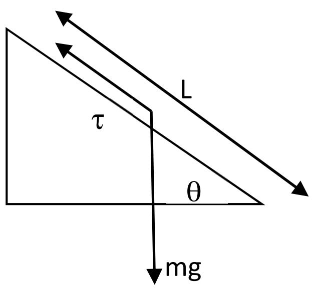


Fig. 3. Definition of bed friction stress

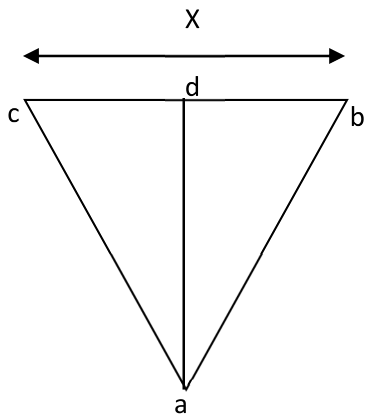
Fig. 3 is showing bed friction stress 'τ' of a stream length 'L' with mass of water 'm' under gravitational pull 'g' on a slope 'θ'.

$\rho mgSin\theta=\tau PL$ Where ρ= co-efficient of friction
$\rho ALgSin\theta=\tau PL$ {m=AL}
Or, $\tau=\rho g(A/P)\sin\theta${as m=AL}
Or, $\tau=\rho g(R_h)\sin\theta${as A/P= R_h}

Average velocity (V) is proportional to $R_h^{2/3}$ and Quantity of flow Q=VA= (1/n) $AR_h^{2/3}\sqrt{slope}$. Therefore, lesser the 'P', lesser the 'τ' and higher is the R_h.

4. THE MOST EFFICIENT TRIANGULAR CROSS-SECTION

Let cross-sectional shape be like an Equilateral Triangle (fig. 4. i), like wide Isosceles triangle (Figs. 4. ii) and like narrow Isosceles triangle (figs.4.iii). In all cases, cross-sectional area is 'A' and length of wetted perimeter is represented by 2ac, 2pr and 2mo respectively. Type i. channel has a width of x, type ii. channel has a width of 2x and type iii. channel has a width x/2. Now let ratio of wetted perimeters of three types of channel be determined.



Equilateral Triangle

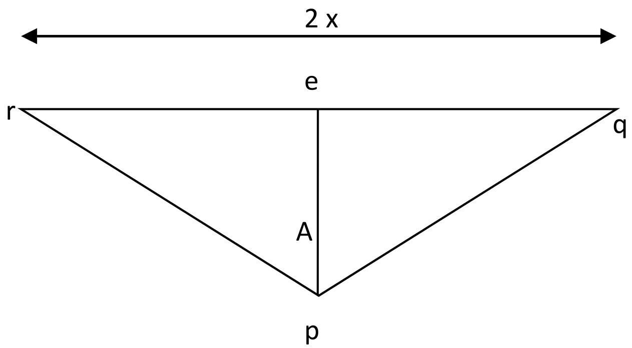
Fig. 4. (i)ab=bc=ca, Equilateral Triangular cross-sectional form with constant area but minimum wetted perimeter

However, if triangular cross-sectional form is the case, then ideal shape is equilateral triangular with 'best convayence characteristics'. From following calculation, it can easily be derived that, isolateral triangular cross-sectional form has the least hydraulic perieter with maximum effiency.

Suppose, area of the equilateral Δ abc (Figs.4.i) is A.

Therefore, $A = \frac{\sqrt{3}}{4}(ab)^2$

Or, $A = \frac{\sqrt{3}}{4}x^2$ [as ab = x]



Isosceles triangle

Fig. 4. (ii)pr=pq and qr=2bc, Isosceles Triangular cross-sectional form with constant area but more wetted perimeter

In Δ pqr, area = A and pq=pr

Area of Δ pqr = ½ qr ×ep

= ½ 2x × ep [as qr= 2x]

= x × ep

Therefore, x × ep $=\frac{\sqrt{3}}{4}x^2$

ep $= \frac{\sqrt{3}}{4}x$

In Δ per, $(pr)^2 = (ep)^2 + (er)^2$

$= (ep)^2 + x^2$ [as er= x]

$= (\frac{\sqrt{3}}{4}\ x)2\ +\ x2$ [as ep $= \frac{\sqrt{3}}{4}x$]

$= \frac{3}{16}x^2 + x^2$

$= (3x^2+16x^2) \div 16$

$= (3x^2+16x^2) \div 16$

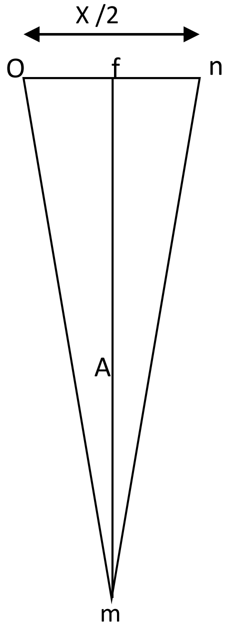
pr $= \frac{\sqrt{19}}{4}x$

$= \frac{\sqrt{19}}{4}x$

$= \frac{4.3}{4}x$

4pr = 4.3x

$\frac{x}{4} = \frac{pr}{4.3}$ (i)



Isosceles triangle

Fig. 4. (iii)pr=pq and qr=2bc, Isosceles Triangular cross-sectional form with constant area but most wetted perimeter

In Δ mno, mn = mo

Area of Δ mno = ½ on × fm

$= \frac{1}{2} \times \frac{x}{2} \times fm$

$= \frac{x}{4} \times fm$

Now, $\frac{x}{4} \times fm = \frac{\sqrt{3}}{4} x^2$ [as Area of all three triangles are equal]

$fm = \sqrt{3}\, x$

In Δ fmo, $(om)^2 = (fm)^2 + (fo)^2$

$= (fm)^2 + (x/4)^2$ [as fo= x/4]

$= (\sqrt{3}\, x)^2 + (x/4)^2$ [as fm= √3x]

$= 3x^2 + x^2/16$

$= (48x^2 + x^2) \div 16$

$= 49x^2/16$

$= 49x^2/16$

om = 7x/4

$$\frac{x}{4} = \frac{om}{7} \quad \text{(ii)}$$

Therefore, $\frac{x}{4} = \frac{pr}{4.3} = \frac{om}{7} = k$ (iii)

x=4k, pr=4.3k and om=7k

Therefore, ratio of wetted perimeter of channel types i:ii:iii = 8:8.6:14.

So, hydraulic radius with given cross-sectional areaof wide V-shaped and narrow V-shaped triangles are smaller than that of an equilateral triangle. So, ideal cross-sectional form of a river is either semicircular or equilateral triangular.

5. IDEAL FORM RATIOS FOR TRIANGULAR CROSS-SECTIONAL FORM

5.1. Ideal width (ẃ) : width index (I_w)

Ideal width provides tool to compare width of a cross-section of a channel with given area to that of the ideal width which the channel tries to attain for best conveyance. Width index (I_w) is defined as I_w = w/ ẃ.

I_w = w/ẃ.

Here, ẃ is derived as follows-

ẃ = x, [x is the length of a side of an equilateral triangle]

Now, if the cross-sectional area of the concerned channel is 'A', then

$A = \frac{\sqrt{3}}{4}x^2$

Or, $x^2 = 4A/\sqrt{3}$

Or, $x^2 = 2.3094A$

Or, $x = 1.52A$

Or, ẃ $= 1.52\sqrt{A}$ (1)

5.2. Ideal mean depth (Ď): depth index (I_d)

Ideal mean depth provides tool to compare mean depth (d) of a cross-section of a channel with given area to that of the ideal mean depth (Ď) which the channel tries to attain for best conveyance.

Depth index (I_d) is defined as I_d = d / Ď.

I_d = d / Ď.

Here, Ď is derived as follows-

Ď $= A/1.52\sqrt{A}$

Or, Ď $= \sqrt{A}/1.52$ (2)

5.3. Ideal form ratio (C_fI)

C_fI = ẃ / Ď

Or, $C_fI = \frac{1.52\sqrt{A}}{\sqrt{A}/1.52}$

Or, $C_fI = 2.31$ (3)

If the channel is like triangular in shape and if the C_fI = 2.31, the cross-sectional form is equilateral triangular which represents the form of middle course or mature stage. Lesser the value, narrower the shape and implies the channel of upper course or youth stage. If the value is greater than 2.31, it implies the wider v-shaped channel of lower reach or old stage.

6. CONCLUSIONS

The relative rates of increase of width and depth are functions of channel shape. But mere width: mean-depth ratio (w/d) does not define cross-sectional shape (Hey, 1978) even though it is a widely used index. So to have comparison, instead of simple width to mean-depth ratio, Ideal form ratio-3 ($C_f I$ = 2.31) may give better explanation. Even, with a given area of channel cross-section, how far its width and depth are deviated from the ideal value can be determined by Ideal form ratio-1 (ẃ $=1.52\sqrt{A}$) andIdeal form ratio-2 (Ď = √A/*1.52*).

Acknowledgement

No word is enough to express my gratitude to my teacher Prof.Sandip Kumar Choudhury, worthy but far away from lime light, and who enabled me to think about these numbers. Ideal form ratios, named after him, is my hearty homage to him. I acknowledge my friends from different disciplines and my students who helped to materialize this paper.

R E F E R E N C E S

1. Clifton, H.E. (1976). Wave-Foirmed Sedimentary Structure- A Conceptual Model. In D.R.L, *Beach And Near Shore Sedimentation,* (Pp. 126-148). Society Of Ecxonoic Paleontologistits, Special Publication.
2. Chaw, V.T. (1959). *Open Channel Hydraulics.* New York: Mcgraw Hill.
3. Crickmay, H.C. (1974). *The Work Of The River.* London : Mcmillan Press Ltd.
4. Das, B.C. (2013). *Changes And Deterioration Of The Course Of River Jalangi And Its Impact On The People Living On Its Banks, Nadia, West Bengal: Ph.D. Thesis.* Kolkata: University Of Calcutta.
5. Fischenich, C. (2000). Robert Manning: A Historical Perspective. *TN-EMRRP-SR-10, US Army Corps Of Engineers Research And Development Center, Environmental Laboratory.*
6. Harvey, A.M. (1975). Somring Streamse Aspects Of The Relations Between Channel Characteristics And Riffle Spacing In Meand. *American Journal Of Science*, *275*, 470-478.
7. Hey, R.D. (1978). Determinate Hydraulic Geometry Of River Channels.,. *Journal Of The Hydraulics Division*, *104, HY4*, 365-379.
8. Kennedy, B.A. (1976). Valley-Side Slopes And Climate. In Derbyshire. E, *Geoorphology And Climate* Chichester: Wiley. (Pp. 171-202).
9. Knighton, A.D. (1981). A Symmetry O F River Channel Cross-Sections: Part 1. Quantitative Indices. *Earth Surface Processes And Land-Forms*, *VI*, P-581-588.
10. Knighton, A.D. (1982). Asymmetry O F River Channel Cross-Sections: Part 2. Mode Of Development And Local Variation. *Earth Surface Processes And Land-Forms*, *VII*, P-117-131.
11. Knighton, A.D. (1998). *Fluvial Forms And Processes A New Perspective.* London: Arnold.
12. Kranck, K. (1972). Tidal Current Control Of Sediment Distribution In Northumberland Strait, Aritime Provinces. *Journal Of Sedimentary Petrology*, *42*, 596-601.

13. Lane, E.W. (1955). Design Of Stable Alluvial Channels.. *Transaction Of The American Society Of Civil Engineers*, *120 (2776)*, P- 1234-1260.
14. Leopold, L.B. (1966). River Meanders. *Scientific American*, *214(6)*, 60.
15. Leopold, L.B. Jr, Maddock, T. (1953). The Hydraulic Geometry Of Stream Channels And Some Physioghaphic Iplications. *United States Geological Survey Professional Paper, 500A*, *252*.
16. Leopold, L.P., Miller, J. (1956). Ephemeral Streams-Hydraulic Factors And Their Relation To The Drainage Net. *United States Geological Survey Professional Paper,*, *282A*.
17. Leopold, L.B., Wolman, M.G. (1969). *Fluvial Processes In Geoorphology.* New Delhi: Eurasia Publishing House Pvt. Ltd. Reprint.
18. Leopold, L.B. And Wolman, M.G. (1981). River Meanders. *Geological Society Of America Bulletin*, *71*, 769-794.
19. Lewis, L.A. (1969). Some Fluvial Geomorphic Characteristics Of The Manati Basin, Puertorico. *Annals Of The Association Of American Geographers*, *59*, 280-293.
20. Manning, R. (1891). On The Flow Of Water In Open Channels And Pipes. *Transactions Of The Institute Of Civil Engineers Of Ireland*, 39-42.
21. Miller, L.L. (1956). Ephemeral Streams- Hydraulic Factors And Their Relation To The Drainage Net. *United States Geological Survey Professional Paper*, *282A*.
22. Milne, J.A. (1979). *The Morphological Relationships Of Bends In Confined Stream Channels In Upland Britain* (Vol. Geographical Approceaches To Fluvial Processes). (A. F. Pitty, Ed.) Norwich: Geobooks.
23. Schumm, S.A. (1960). The Shape Of Alluvial Channels In Relation To Sediment Type, Professional Paper. *United States Geological Survey*, *352B*, 17-30.
24. Sen, P.K. (1993). *Geomorphological Analysis Of Drainage Basins.* Burdwan: The University Of Burdwan.
25. Sharp, R.P. (1963). Wind Ripples. *Journal Opf Geology*, *71*, 617-636.
26. Simon, A. (1992). Energy, Time And Channel Evolution In Catastrophically Disturbed Fluvial Systems. (P. J. H, Ed.) *5*, 345-372.
27. Simon, A.E., Darby, S. (1997). Process-Form Interactions In Unstable Sand Bed River Channels: A Nuerical Modelling Approach. *Geomorphology*, *21*, 85-106.
28. Simson, A.J. (2003). Measureent And Analysis Of Alluvial Channel Form. In K. G. H, *Tools In Geoorphology* (Pp. 1291-346). Chichester: Wiley.
29. Tanner, W.F. (1967). Ripple Mark Indices And Their Uses. *Sedimentology*, *9*, 89-104.
30. Wilcock, D.N. (1971). Investigation Into The Relations Between Bedload Transport And Channel Shape. *Bulletin Of The Geological Society Of America*, *82*, 2159-2176.
31. Wolman, M.G. (1955). The Natural Channel Of Brandywine Creek, Pennsylvania. *United States Geological Survey Professional Paper*, 271.