IN SEARCH OF IDEAL FORM- RATIO OF TRIANGULAR CHANNEL

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ABSTRACT. – In Search of Ideal Form-Ratio of Triangular Channel. Cross-sectional form of a natural channel is a two dimensional variable which is thoroughly studied by scholars from different fields on natural sciences like hydrology, geology, geomorphology, etc. Average river channels tend to develop their channel-cross sectional form in a way to produce an approximate equilibrium between the channel and the water and sediment it transport. But how far it is deviated from the ideal cross-sectional form can only be determined by knowing the ideal form which was calculated by Hickin for rectangular channel. This ideal cross-sectional form of ‘maximum efficiency’ is virtually a theoretical one and attaining of which the river transports its water and load with least friction with its bed. ‘Ideal form ratio’ provides numerical tools for triangular channel to determine the degree of deviation of a cross-sectional form from that of an ideal one.

Keywords: channel asymmetry, cross-sectional area, hydraulic radius, ideal form ratio, wetted perimeter.

1. INTRODUCTION

The cross-sectional form of natural stream channel is characteristically irregular in outline and locally variable (Knighton, 1998). Commonly two groups of aspects are considered to describe channel cross-sectional form – i) channel size and ii) channel shape. The first group includes width (w), mean depth (d), cross-sectional area (A), wetted perimeter (P), hydraulic radius (R), maximum depth (dmax), and bed width (wb) while the second group of variables are width: depth ratio (w/d), maximum depth: mean depth ratio (dmax/d), channel asymmetry, bank slope (∅) and bed topography. About 9/10 meandering channels cross-sections are asymmetric (Leopold and Wolman, 1960) and asymmetry is assumed to be found at cross-overs (Knighton, 1981) although cross-sectional asymmetry is found in so-called straight channels (Einstine and Shen, 1964; Keller, 1972). For the study of channel’s cross-sectional form, it is indispensable to know the ratio and Hickin (2004) calculated it for rectangular channel. The present paper is related to the cross-sectional shape of triangular channel and aims to formulate indices on channel form to determine the magnitude of deviation of a cross-sectional shape from the ideal one.

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2. EVALUATION OF EXISTING MEASURES

Asymmetry of natural features has been studied from the viewpoint of processes of action (Kranck, 1972; Clifton, 1976) and shape (Sharp, 1963; Tanner, 1967; Kennedy, 1976). The cross-sectional form of a river is primarily adjusted by bank erosion and lateral channel migration (Simon and Castro, 2004) and represents dominant measures of channel response (Simon, 1992; Simon and Darby, 1997). Channel shape is rarely explained in a single measure like width: depth ratio (w/d), maximum depth: mean depth ratio (d_{max}/d), channel asymmetry, bank slope (∅), etc. **Width: mean-depth ratio** (w/d) does not define cross-sectional shape (Hey, 1978) yet it is a widely used index. **Maximum depth: mean depth ratio** (d_{max}/d) is another measure which gives no better expression than that of Width: mean-depth ratio (w/d).

Form ratio w/d or w/d_{max} (Schumm, 1960) and section asymmetry a_{l}/a_{r} (Milne, 1979) regarding cross-sectional form give an absolute measure. These measures do not tell about the quality of the ratio i.e. how far the value is deviated from ideal one. That is why ideal value of form ratio of triangular channels is to be known.

2.1. Channel asymmetry: Channel asymmetry is a measure of the degree of asymmetry of the two halves of the cross-sectional area demarcated by the central line drawn vertically from the mid-width point of the channel. Three channel asymmetry indices were defined (Fig. 1) by Knighton (1981).

![Fig. 1. Definitions of Asymmetry Indices](source: Knighton (1998))

1. A* = (A_{r} - A_{l}) / A

Where A_{r} and A_{l} are the cross-sectional areas, respectively, to the right and left of the channel centerline, A (= A_{r} + A_{l}) is the total area. Zero (0) value of the index indicate perfect symmetrical channel cross-sectional form while +1 and -1 indicates the maximum limits of asymmetry to the left and right respectively.

2. A_{l} = (2x d_{max}) / A

The equation is derived as (2x/w) (d_{max}/d). Now, w × d is replaced by A. x is the distance from the centerline to the centroid of maximum depth, d_{max} is maximum depth, and d is mean depth.
3. \[ A_2 = 2x(d_{\text{max}} - d) / A \]

The equation is simplified form of \((2x/w)(d_{\text{max}} - d/d)\). Here depth of the channel is given appropriate weightage. To show the comparison amongst asymmetry in terms of width and depth of channels of equal area, Knighton (1981) used the term 'vertical asymmetry'. For knowing degree of vertical asymmetry, calculation of ideal depth of a channel of given cross-sectional area is precondition. Ideal form ratio provides that tool.

2.2. Bank slopes: Bank slopes measured in degrees are essentially concave in nature with rare variations like rectilinear or convex. Banks of Type- C and Type- B (Fig. 2) channels (Knighton, 1998) are rectilinear (Knighton 1998) while Type- A is more concave in nature. Degree of curvature (concavity or convexity) of a bank slope may be measured by the ratio of arc length to the angle in between lines of radii joining two ends of that arc. There is a opposite relation between the ratio and the degree of curvature of slope.

\[ \text{Fig. 2. Different Type of Cross-sectional form} \]
\[ \text{Source: Knighton (1998)} \]

2.3. Bed Topography: The last but not the least consideration of the channel cross-sectional form is its bed topography. Riffles-pool, step-pool, ripple, bar, shoal etc gives every channel a unique cross-sectional form which is beyond the capacity of a uniform measure to explain the shape as it is.

3. IDEAL CROSS-SECTIONAL FORM

The conventional belief of the v-shaped cross-sectional form of the rivers is far from the reality (Sen, 1993). Circular (Leopold et al, 1969) and parabolic (Lane, 1955) forms are also theoretical. Rather trapezoidal form represents the reality (Sen, 1993). But all these forms, whether theoretical or practical, are not obvious for all the channels or the entire reach of the same channel. The straight course of a river is impossible (Leopold and Langbein, 1966) which makes another impossibility of uniformity of cross-sectional form of the channel. Width increases faster than depth in downstream and cross-sectional form becomes increasingly rectangular (Sen, 1993). But sometimes the opposite is also the reality (Knighton, 1998; Das, 2013).
B. C. DAS

The conditions of efficiency of the cross-sectional characteristics of the channels are closely related to their capacity for allowance of maximum flow. Maximum flow (water + sediment load) is only possible when the cross-sectional form attains the semi-circular or parabolic shape (Knighton, 1998) or equilateral-triangular. These shapes generate the minimum turbulence and shear stress hence the 'most efficient' channels. Thus this ideal condition of channel form is considered as the 'best conveyance characteristics' (Crickmay, 1974). Arcs of circles and parabolic curves may be simple devices for the explanation of the channel cross-sectional form. The relationship between channel form and processes operating in the channels has been studied as hydraulic geometry of the stream channels by Leopold and Maddock (1953), Wolman (1955), Leopold and Miller (1956) and others. They computed cross-sectional forms in terms of mean-depth \( d \) and width \( w \) in terms of hydraulic parameter discharge \( Q \).

\[
\begin{align*}
  w &= aQ^b \\
  d &= cQ^f
\end{align*}
\]

Different average exponent values for \( b \) and \( f \) of different rivers have been calculated by Leopold and Maddock (1953), Wolman (1955), Leopold and Miller, (1956), Lewis (1969), Wilcock (1971) and Harvey (1975).

Manning (1891) flow resistance equation \( v = k \left( R^{2/3}S^{1/2} \right)/n \) (Knighton, 1998) or \( v = (1/n) R^{2/3}S^{1/2} \) (Simon and Castro, 2003) suggests that channel form and flow resistance determine the velocity of a river. To built up his equation, Manning utilised seven different open channel flow equations and the findings were field tested (Chaw, 1959; Fischenich, 2000). In this equation, \( v \) = velocity of flow, \( R \) or \( R_h \) = hydraulic radius of the channel, \( S \) or \( S_b \) = slope, and \( n \) = manning resistance co-efficient. With given volume, velocity is proportional to hydraulic radius \( R^{2/3} \) and slope \( S^{1/2} \) but inversely proportional to shear resistance. Valley slope and floodplain slope is generally greater than channel slope (Simon and Castro, 2003). If all other variables of the equation are constant, the equation implies that the velocity over flood plain is greater than in the channel. But reality is opposite because of- i) Roughness, hence shear strength over floodplain is much greater than in the channel; ii) Hydraulic radius is lesser over floodplain.

\[ \text{Fig. 3. Definition of bed friction stress} \]

\[ \tau \]

Fig. 3 is showing bed friction stress ‘\( \tau \)’ of a stream length ‘L’ with mass of water ‘m’ under gravitational pull ‘g’ on a slope ‘\( \theta \)’.
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ρmgSinθ=τPL Where ρ= co-efficient of friction
ρALgSinθ=τPL (m=AL)
Or, τ=ρg(A/P)sinθ(as m=AL)
Or, τ=ρg(Rh)sinθ(as A/P= Rh)

Average velocity (V) is proportional to $R_h^{2/3}$ and Quantity of flow $Q=VA= (1/n)ARh^{2/3}\sqrt{\text{slope}}$. Therefore, lesser the ‘P’, lesser the ‘τ’ and higher is the $R_h$.

4. THE MOST EFFICIENT TRIANGULAR CROSS-SECTION

Let cross-sectional shape be like an Equilateral Triangle (fig. 4. i), like wide Isosceles triangle (Figs. 4. ii) and like narrow Isosceles triangle (figs.4.iii). In all cases, cross-sectional area is ‘A’ and length of wetted perimeter is represented by 2ac, 2pr and 2mo respectively. Type i. channel has a width of x, type ii. channel has a width of 2x and type iii. channel has a width x/2. Now let ratio of wetted perimeters of three types of channel be determined.

![Equilateral Triangle](image)

**Fig. 4. (i)**ab=bc=ca, Equilateral Triangular cross-sectional form with constant area but minimum wetted perimeter

However, if triangular cross-sectional form is the case, then ideal shape is equilateral triangular with ‘best conveyance characteristics’. From following calculation, it can easily be derived that, isolateral triangular cross-sectional form has the least hydraulic perimeter with maximum efficiency.

Suppose, area of the equilateral Δ abc (Figs.4.i) is A.

Therefore, $A = \frac{\sqrt{3}}{4}(ab)^2$

Or, $A = \frac{\sqrt{3}}{4}x^2$ [as ab = x]
Fig. 4. (ii) $pr=pq$ and $qr=2bc$, Isosceles Triangular cross-sectional form with constant area but more wetted perimeter

In $\triangle pqr$, area $= A$ and $pq=pr$

Area of $\triangle pqr = \frac{1}{2} qr \times ep$

$= \frac{1}{2} \times 2x \times ep$ \quad [as $qr=2x$]

$= x \times ep$

Therefore, $x \times ep = \frac{\sqrt{3}}{4}x^2$

$ep = \frac{\sqrt{3}}{4}x$

In $\triangle per$, $(pr)^2 = (ep)^2 + (er)^2$

$= (ep)^2 + x^2$ \quad [as $er=x$]

$= \left(\frac{\sqrt{3}}{4}x\right)^2 + x^2$ [as $ep = \frac{\sqrt{3}}{4}x$]

$= \frac{3}{16}x^2 + x^2$

$= (3x^2 + 16x^2) \div 16$

$= (3x^2 + 16x^2) \div 16$

$pr = \frac{\sqrt{19}}{4}x$

$= \frac{\sqrt{19}}{4}x$

$= \frac{4.3}{4}x$

$4pr = 4.3x$

$x = \frac{pr}{4.3}$ \quad (i)
In Δ mno, mn = mo

Area of Δ mno = \( \frac{1}{2} \times on \times fm \)
\[ = \frac{1}{2} \times \frac{x}{2} \times fm \]
\[ = \frac{x}{4} \times fm \]

Now, \( \frac{x}{4} \times fm = \frac{\sqrt{3} x^2}{4} \) [as Area of all three triangles are equal]

In Δ fmo, \((om)^2 = (fm)^2 + (fo)^2\)
\[ = (\sqrt{3} x)^2 + (x/4)^2 \] [as fo = x/4]
\[ = 3x^2 + x^2/16 \]
\[ = (48x^2 + x^2)/16 \]
\[ = 49x^2/16 \]
\[ om = 7x/4 \]

\[ \frac{x}{4} = \frac{om}{pr} \] \( \quad \text{(ii)} \)

Therefore, \( \frac{x}{4} = \frac{7}{4.3} = \frac{om}{pr} = k \) \( \quad \text{(iii)} \)

x=4k, pr=4.3k and om=7k
Therefore, ratio of wetted perimeter of channel types i:ii:iii = 8:8.6:14.
So, hydraulic radius with given cross-sectional area of wide V-shaped and narrow V-shaped triangles are smaller than that of an equilateral triangle. So, ideal cross-sectional form of a river is either semicircular or equilateral triangular.

5. IDEAL FORM RATIOS FOR TRIANGULAR CROSS-SECTIONAL FORM

5.1. Ideal width (ẃ) : width index (Iw)

Ideal width provides tool to compare width of a cross-section of a channel with given area to that of the ideal width which the channel tries to attain for best conveyance. Width index (Iw) is defined as \( I_w = \frac{w}{\bar{w}} \).

\( \bar{w} \) is derived as follows-

\[ w = \frac{\sqrt{A}}{1.52} \]

Now, if the cross-sectional area of the concerned channel is ‘A’, then

5.2. Ideal mean depth (\( \bar{D} \)): depth index (Id)

Ideal mean depth provides tool to compare mean depth (d) of a cross-section of a channel with given area to that of the ideal mean depth (\( \bar{D} \)) which the channel tries to attain for best conveyance.

Depth index (Id) is defined as \( I_d = \frac{d}{\bar{D}} \).

\( \bar{D} \) is derived as follows-

\[ \bar{D} = \frac{A}{\sqrt{A/1.52}} \]

5.3. Ideal form ratio (CfI)

\[ CfI = \frac{\bar{w}}{\bar{D}} \]

If the channel is like triangular in shape and if the \( CfI = 2.31 \), the cross-sectional form is equilateral triangular which represents the form of middle course or mature stage. Lesser the value, narrower the shape and implies the channel of upper course or youth stage. If the value is greater than 2.31, it implies the wider v-shaped channel of lower reach or old stage.
6. CONCLUSIONS

The relative rates of increase of width and depth are functions of channel shape. But mere width: mean-depth ratio (w/d) does not define cross-sectional shape (Hey, 1978) even though it is a widely used index. So to have comparison, instead of simple width to mean-depth ratio, Ideal form ratio-3 ($C_f = 2.31$) may give better explanation. Even, with a given area of channel cross-section, how far its width and depth are deviated from the ideal value can be determined by Ideal form ratio-1 ($\bar{w} = 1.52\sqrt{A}$) and Ideal form ratio-2 ($\bar{D} = \sqrt{A/1.52}$).

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REFERENCES


86